Terminating and Repeating Decimals

What You'll Learn
Scan the lesson. Predict two things you will learn about terminating and repeating decimals.

Essential Question
HOW are integers and absolute value used in real-world situations?

Vocabulary
terminating decimal
repeating decimal
bar notation
rational number

Vocabulary Start-Up
Any number that can be written as a fraction is called a rational number. Every rational number can be written as either a terminating decimal or a repeating decimal.

Draw lines from each word to its matching statement.

- terminating decimal
- the decimal form of a rational number; 0.33333...
- repeating decimal
- the decimal form of a rational number which has a repeating digit of zero; 06.25

Real-World Link
Party Favors Jude is buying fruit snacks for party favors. He asks the cashier for a half pound of fruit snacks.

1. Express one half as a fraction.

2. Write the decimal that represents half a pound.

3. Suppose Jude wanted to buy one third of a pound. What decimal would the scale show?
Rational Numbers

Words: Rational numbers can be written as fractions.
Algebra: $\frac{a}{b}$, where $a$ and $b$ are integers and $b \neq 0$.
Model:

Fractions, terminating and repeating decimals, percents, and integers are all rational numbers. Every rational number can be expressed as a decimal by dividing the numerator by the denominator.

To indicate the number pattern that repeats indefinitely, use bar notation. **Bar notation** is a bar placed over the digits that repeat.

<table>
<thead>
<tr>
<th>Rational Number</th>
<th>Repeating Decimal</th>
<th>Terminating Decimal</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{3}{10}$</td>
<td>0.300...</td>
<td>0.3</td>
</tr>
<tr>
<td>$\frac{4}{5}$</td>
<td>0.800...</td>
<td>0.8</td>
</tr>
<tr>
<td>$\frac{5}{6}$</td>
<td>0.833...</td>
<td>does not terminate</td>
</tr>
</tbody>
</table>

**Example**

1. Write $\frac{5}{12}$ as a decimal.

   0.4166

   $12)5.000$
   $\frac{-48}{20}$
   $\frac{-12}{80}$
   $\frac{-72}{8}$

   So, $\frac{5}{12} = 0.4166...$ or $0.41\bar{6}$. 
Got It? Do these problems to find out.

Write each fraction as a decimal. Use bar notation if necessary.

a. \( \frac{1}{6} \)
   
   \[ 1 \div 6 = 0.1\overline{6} \]

b. \( \frac{8}{9} \)
   
   \[ 8 \div 9 = 0.\overline{8} \]

c. \( \frac{2}{11} \)
   
   \[ 2 \div 11 = 0.\overline{18} \]

Write a Negative Fraction as a Decimal

When writing negative fractions as decimals, the process is the same. Divide as with positive fractions. Write the negative sign in front of the decimal.

Examples

2. Write \( -\frac{2}{9} \) as a decimal.

   \[
   \begin{array}{c}
   \hline
   2.000 \\
   9) \hline
   \hline
   18 \\
   18 \\
   \hline
   2 \\
   \end{array}
   \]

   \[ \text{So, } -\frac{2}{9} = -0.222... \text{ or } -0.\overline{2}. \]

3. Write \( -2\frac{2}{3} \) as a decimal.

   \[ -2\frac{2}{3} \text{ can be rewritten as } -\frac{8}{3}. \]

   The mixed number \(-2\frac{2}{3}\) can be written as \(-2.\overline{6}\).

Got It? Do these problems to find out.

Write each fraction as a decimal. Use bar notation if necessary.

d. \( -\frac{1}{4} \)

   \[ -0.25 \]

e. \( -\frac{5}{6} \)

   \[ -0.\overline{83} \]

f. \( -2\frac{1}{6} \) or \( -\frac{13}{6} \)

   \[ -2.1\overline{6} \]
Example

4. Frankie made 34 out of 44 free throws this season. To the nearest thousandth, what is his free-throw average?

Using a calculator, divide 34 by 44.

\[ \frac{34}{44} \]

To the nearest thousandth, his free-throw average is 0.773.

Got It? Do this problem to find out.

g. Of nine students surveyed, four said they prefer exercising in the morning rather than in the evening. Express this fraction as a decimal. Use bar notation if necessary.

Guided Practice

Write each fraction as a decimal. Use bar notation if necessary. (Examples 1–3)

1. \( \frac{7}{9} = 0.\overline{7} \)

2. \( \frac{1}{33} = 0.0\overline{3} \)

3. \( -\frac{5}{6} = -2.8\overline{3} \)

4. \( \frac{10}{15} = 0.\overline{6} \)

5. \( \frac{4}{5} = -0.8 \)

6. \( 1\frac{5}{9} = 1.\overline{5} \)

7. Dana bought \( \frac{2}{3} \) yard of fabric to make a new purse. Write the amount of fabric she used as a decimal. (Example 4)

8. Building on the Essential Question How are repeating decimals used in real-world situations?
Write each fraction as a decimal. Use bar notation if necessary.

(Examples 1–3)

1. \( \frac{7}{15} = \)  

2. \( \frac{8}{18} = \)  

3. \( \frac{8}{12} = \)  

4. \( -\frac{6}{7} = -0.857142 \)  

5. \( 3\frac{15}{44} = \)  

6. \( -2\frac{5}{22} = \)  

7. Sarafina had 34 out of 99 hits when she was at bat during the softball season. What was her batting average? (Example 4)

8. Shiv and his friends ate 3\( \frac{4}{6} \) pizzas. Write this amount as a decimal. (Example 4)

Write each decimal as a fraction or mixed number in simplest form.

9. \(-0.9 = \)  

10. \(-0.85 = \)  

11. \(-3.8 = \)  

Evaluate each expression.

12. \( |-2.3| = \)  

13. \( \left| \frac{4}{13} \right| = \)  

14. \( |-8\frac{7}{11}| = \)  

15 STEM There are over 2,700 species of snakes in the world. Over 600 species are venomous. Write the fraction of species that are not venomous as a decimal.

16. Justify Conclusions The ratio of the circumference of a circle to its diameter is represented by the number \( \pi \). The number \( \pi \) is a decimal that does not repeat. The fraction \( \frac{22}{7} \) is sometimes used as an estimate of \( \pi \). Is \( \frac{22}{7} \) a repeating decimal? Explain.